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ROOTS: THE SEARCH FOR ZEROS OF THE CHARACTERISTIC FUNCTION FOR THE SOLID ROD

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US ARMY ARMAMENT RESEARCH AND DEVELOPMENT COMMAND
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The method of calculation of roots of the characteristic equation for the solid rod is presented. Difficulties with Newton's method are shown to be circumvented by application of a modified method of bisections.

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I. INTRODUCTION

In the analysis of stresses in gun tubes, particular difficulties are encountered near surface boundaries such as at the muzzle, or near discontinuities in stress loading such as at the rotating band of the projectile. The analytic description of stress behavior in such regions is in the class of problems termed end stresses in semi-infinite cylinders. Currently, the Ballistic Research Laboratory (BRL) is supporting work in this area through contract* with Dr. David A. Spence of Oxford University and the Imperial College, U.K.

The research discussed in this report was conducted in support of Dr. Spence's work. Specifically, in view of the existence of BRL'S Bessel function subroutine[1] and experience in computing characteristic roots for the hollow cylinder, Dr. Spence requested the BRL to provide him with a tabulation of the first 1000 roots of the characteristic equation for a range of values of Poisson's ratio.

II. THE CHARACTERISTIC FUNCTION FOR THE SOLID ROD

Cauchy's method of characteristics was applied by V.K. Prokopov[2] to obtain the characteristic equation for a hollow cylinder with stress-free surface. A similar process for the geometry of the semi-infinite solid rod with a stress-free surface leads to the characteristic equation

European Research Office, U.S. Army, Contract No. DAJA 37-80-C-0192.

[1]*K.L. Zimmerman, A.S. Elder, A.K. Depue, "User's Manual for the BRL Subroutine to Calculate Bessel Functions of Integral Order and Complex Argument," Technical Report ARBRL-TR-02068, Ballistic Research Laboratory, 1978 (AD A056369).*

[2]*V.K. Prokopov, "Equilibrium of an Elastic Axisymmetrically Loaded Thick-Walled Cylinder," Prikadnaya mathematika i mehanika, Vol XIII, 1949, pages 135-144. Institute of Mechanics of the Academy of Sciences, USSR, FTIO Translation No. J-2589, Aberdeen Proving Ground, Maryland, Translation dated 22 August 1967.*

$$\Delta(z) = 0 \tag{1}$$

where

$$z\Delta(z) = \begin{vmatrix} I_0(z) & [z^2 + (2-2\nu)] \quad I_1(z) \\ & \\ I_1(z) & z^2 \left[I_0(z) \right] \end{vmatrix}, \tag{2}$$

I_0 and I_1 are modified Bessel functions of the first kind, of order 0 and 1, respectively, and ν is Poisson's ratio.

The roots of Eq. (1) correspond to the poles of the integrands in the Fourier integrals representing the stresses. The values of z satisfying Eq. (1) are therefore needed for calculations based on residue theory. It should be noted that if Z_0 is one such root, then $\bar{Z}_0$, $-Z_0$, and $-\bar{Z}_0$ are also roots, where $\bar{Z}_0$ denotes the complex conjugate of Z_0. Thus, only roots in the first quadrant need be considered.

To evaluate z $\Delta(z)$, use was made of the BRL Bessel function subroutine[1] in the equation

$$z\Delta(z) = z^2[I_0(z)]^2 - \left(z^2 + (2-2\nu)\right)[I_1\ (z^2)]\ . \tag{3}$$

The initial estimate used for the distance between the zeros along the imaginary axis was π units. Since Newton's method[3] worked very well in the simular problem for the hollow cylinder,[4] it was decided that this method would be used, along with the initial root estimates for the cylinder.

III. THE FAILURE OF NEWTON'S METHOD

Once Newton's method begins to converge, it converges quite rapidly. It is also unfortunately true that once the method begins to diverge, it diverges rapidly. This fact is seen at once from the expression for the error terms.[3] We have

$$E_{k+1} = \frac{-f''(\theta)}{2f'(x_k)}(E_k)^2\ , \tag{4}$$

[3] *R.W. Hamming, Numerical Methods for Scientists and Engineers, McGraw-Hill, Inc., New York, 1973.*

[4] *A.S. Elder, K.P. Zimmerman, "Stresses in a Hollow Elastic Cylinder Produced by a Step Function of Pressure or Shear," Memorandum Report BRL MR 2454, Ballistic Research Laboratory, 1975 (AD A008966).*

where E_k is the error in the current estimate x_k of the zero of the function f, E_{k+1} is the error of the next estimate, and θ is some number between X_k and the true zero X. If $f''(\theta)/(2f'(x_k))$ is nearly constant in the vicinity of the root, then the error at each approximation decreases (or increases) as the square of the previous error.

It is therefore critical that the initial estimates of the zeros be quite good when the function has oscillations, as is the case here. In the previously cited case of the hollow cylinder,[4] at most 9 iterations on Newton's method were required to obtain each of the first five hundred roots, with about 20-digit accuracy. The characteristic equation for the hollow cylinder contains products and cross-products of the four modified Bessel functions I_0, I_1, K_0, K_1, and evidently the presence of the K function tempers the oscillations somewhat. In addition, a rather good asymptotic approximation to the characteristic function was found, providing excellent initial estimates of the zeros.

Such was not the case for the present work with the solid rod. The initial estimates proved to be insufficiently accurate to provide convergence in Newton's method.

IV. THE MODIFIED METHOD OF BISECTION

It was decided that a root search based on the method of bisection[3] might be more fruitful. In simple terms, the method of bisection consists of dividing the region near a zero into a mesh of squares. As a square is traversed in the counterclockwise direction, one evaluates the function, seeking a change in sign of the real and/or imaginary parts. This signifies crossing a zero curve of either the real or the imaginary parts, or both. At this point, the search interval is halved, and the process continues in a direction 90° counterclockwise from the direction just used, until another sign change is encountered.

Because of the analyticity of the function the zeros of which we sought, we did not need to concern ourselves with most of the caveats about the use of bisection techniques. The location of the first several zeros was estimated graphically. This was accomplished by computing functional values in the xy-plane and noting sign changes in the real and imaginary parts. The level curves $U = 0$, $V = 0$, where $U(x,y) = \text{Re}\left\{\Delta(x + 2y)\right\}$ and $V(x,y) = \text{Im}\left\{\Delta (x + 2y)\right\}$ are shown in Figure 1.

From this point, computation proceeded by estimating the roots to be separated by π units in the direction of the imaginary axis. Although somewhat more cumbersome and time-consuming than Newton's method, the method of bisections provided values of z for which $|\Delta(z)|<1.0 \times 10^{-20}$ on the BRL CDC 7600 computer. Graphs of the real and imaginary surfaces in the first and fifth quadrants are shown in Figures 2 and 3, respectively. Detail was provided by alternating color contour in these surface plots.

V. CONCLUSIONS

This analysis has shown the method of bisections to be viable tool for use in the method of characteristics. Sensitivity analysis indicated little change beyond the first 10 roots for Poisson's ratio in the range (0.1, 0.5). This information was forwarded along with the tabulation of roots to Prof. Spence.

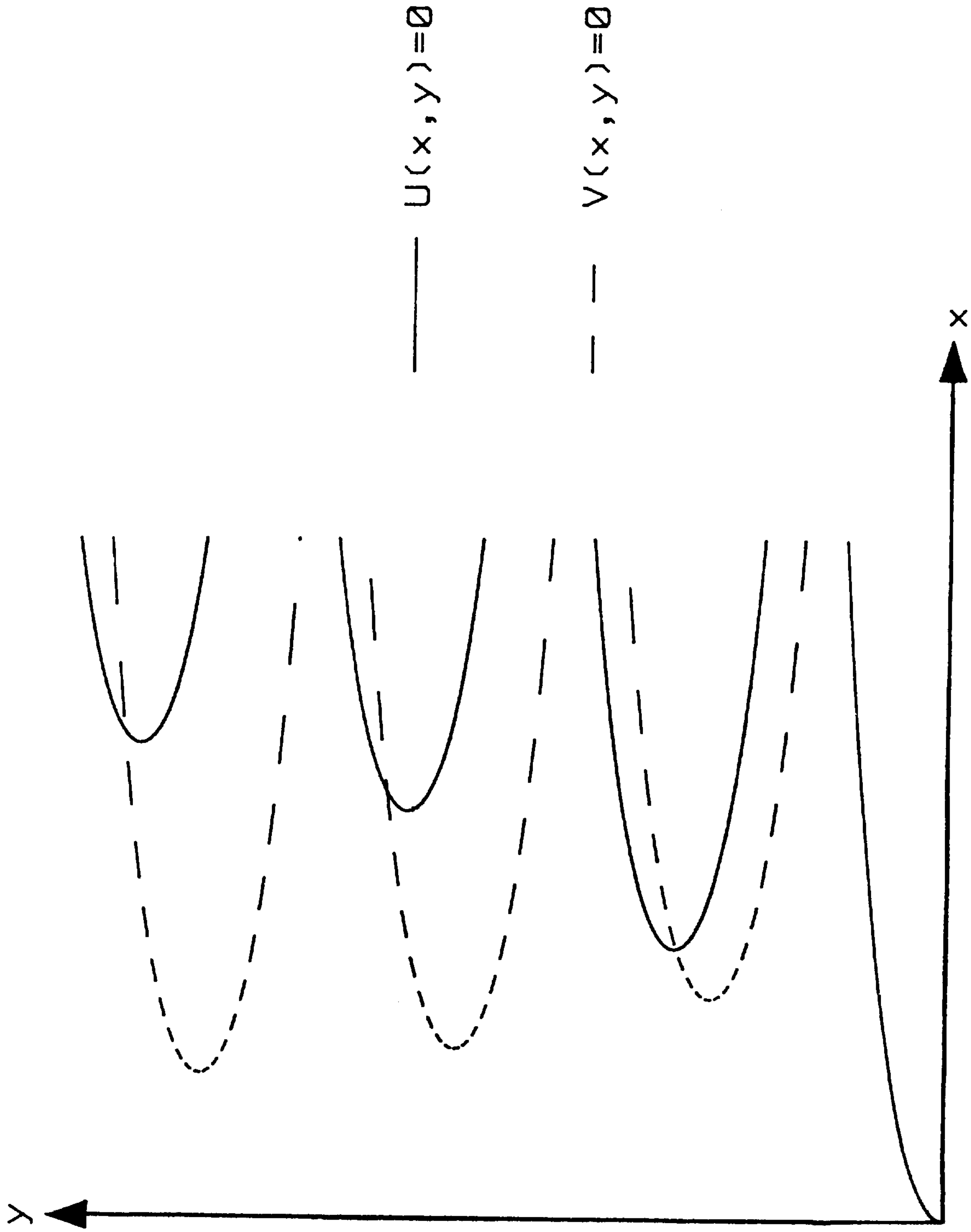

Figure 1. The Real and Imaginary Parts of $\Delta(z)$ in the xy-Plane

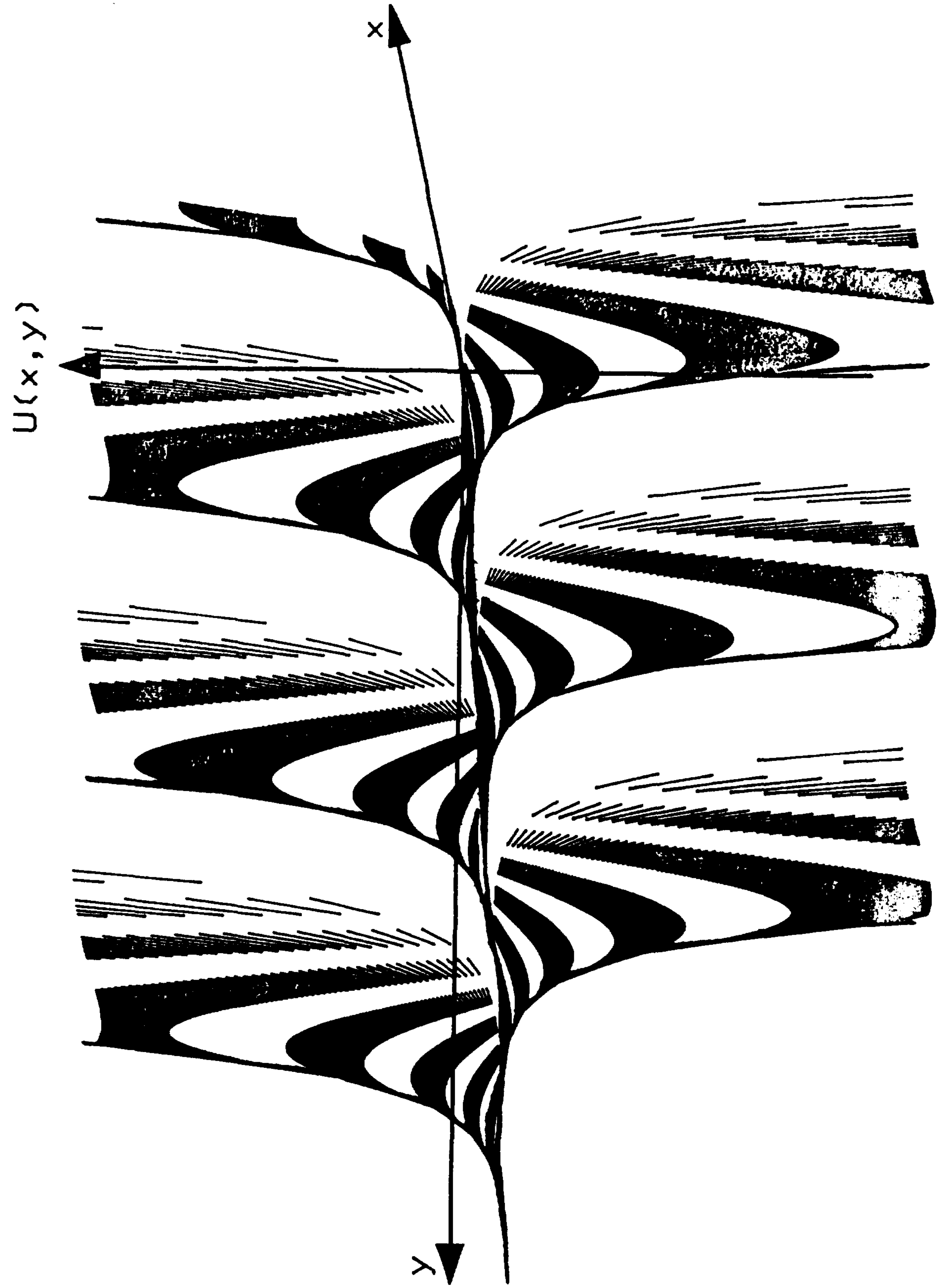

Figure 2. The Real Part of $\Delta(z)$ in the 1st and 5th Quadrants

Figure 3. The Imaginary Part of $\Delta(z)$ in the 1st and 5th Quadrants

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1. K.L. Zimmerman, A.S. Elder, A.K. Depue, "User's Manual for the BRL Subroutine to Calculate Bessel Functions of Integral Order and Complex Argument," Technical Report ARBRL-TR-02068, Ballistic Research Iaboratory, 1978 (AD A056369).

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3. R.W. Hamming, Numerical Methods for Scientists and Engineers, McGraw-Hill, Inc., New York, 1973.

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